

## **Adaptive Control Design and Analysis**

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# **Adaptive Control Design and Analysis**

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**GANG TAO**  
University of Virginia

 **WILEY-INTERSCIENCE**

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*To my parents, my wife Lanlin,  
and my sons Kai and Kwin.*

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# Preface

Adaptive control is becoming popular in many fields of engineering and science as concepts of adaptive systems are becoming more attractive in developing advanced applications. Adaptive control theory is a mature branch of control theories, and there is a vast amount of literature on design and analysis of various adaptive control systems using rigorous methods based on different performance criteria. Adaptive control faces many important challenges, especially in nontraditional applications, such as real-time systems, which do not have precise classical models admissible to existing control designs, or a physiological system with an artificial heart, whose unknown parameters may change at a heart beat rate which is also a controlled variable. To meet the fast growth of adaptive control applications and theory development, a systematic and unified understanding of adaptive control theory is thus needed.

In an effort to introduce such an adaptive control theory, this book presents and analyzes some common and effective adaptive control design approaches, including model reference adaptive control, adaptive pole placement control, and adaptive backstepping control. The book addresses both continuous-time and discrete-time adaptive control designs and their analysis; deals with both single-input, single-output and multi-input, multi-output systems; and employs both state feedback and output feedback. Design and analysis of various adaptive control systems are presented in a systematic and unified framework. The book is a collection of lectures on system modeling and stability, adaptive control formulation and design, stability and robustness analysis, and adaptive system illustration and comparison, aimed at reflecting the state of the art in adaptive control as well as at presenting its fundamentals. It is a comprehensive book which can be used as either an academic textbook or technical reference for graduate students, researchers, engineers, and interested undergraduate students in the fields of engineering, computer science, applied mathematics and others, who have prerequisites in linear systems and

feedback control at the undergraduate level.

In this self-contained book, basic concepts and fundamental principles of adaptive control design and analysis are covered in 10 chapters. As a graduate textbook, it is suitable for a one-semester course: lectures plus reading may cover most of the book without missing essential material. To help in understanding the topics, at the end of each chapter, there are problems related to that chapter's materials as well as technical discussions beyond the covered topics. A separate manual containing solutions to most of these problems is also available. At the end of most chapters, there are also some advanced topics for further study in adaptive control.

Chapter 1 compares different areas of control theory, introduces some basic concepts of adaptive control, and presents some simple adaptive control systems, including direct and indirect adaptive control systems in both continuous and discrete time, as well as an adaptive backstepping control design for a nonlinear system in continuous time.

Chapter 2 presents some fundamentals of dynamic system theory, including system models, system characterizations, signal measures, system stability theory (including Lyapunov stability and input-output operator stability), signal convergence lemmas, and operator norms. In particular, it gives a thorough study of the Lyapunov direct method for stability analysis, some time-varying feedback operator stability properties, several important inequalities for system analysis, some detailed input-output  $L^p$  stability results, various analytical  $L^p$  signal convergence results, some simplified analytical tools for discrete-time system stability, and multivariable operator norms. These results, whose proofs are given in detail and are easy to understand, clarify several important signal and system properties for adaptive control.

Chapter 3 addresses adaptive parameter estimation for a general linear model illustrated by a parametrized linear time-invariant system in either continuous or discrete time. Detailed design and analysis of a normalized gradient algorithm and a normalized least-squares algorithm in either continuous or discrete time are given, including structure, stability, robustness, and convergence of the algorithms. A collection of commonly used robust adaptive laws are presented which ensure robust stability of the adaptive schemes in the presence of modeling errors. An  $L^{1+\alpha}$  ( $\alpha \geq 1$ ) theory is developed for adaptive parameter estimation for a linear model, revealing some important inherent robustness properties of adaptive parameter estimation algorithms.

Chapter 4 develops two types of state feedback adaptive control schemes: for state tracking and for output tracking (and its discrete-time version). For both continuous- and discrete-time systems, adaptive state feedback for output tracking control, based on a simple controller structure under standard model reference adaptive control assumptions, is used as an introduction to adaptive control of general linear systems. Adaptive disturbance rejection under different conditions is addressed in detail; in particular, adaptive output rejection of unmatched input disturbance is developed based on a derived property of linear systems. Another development is a derived parametrization of state feedback using a full- or reduced-order state observer, leading to the commonly used parametrized controller structures with output feedback.

Chapter 5 deals with continuous-time model reference adaptive control using output feedback for output tracking. The key components of model reference adaptive control theory—*a priori* plant knowledge, controller structure, plant-model matching, adaptive laws, stability, robustness, and robust adaptation—are addressed in a comprehensive formulation and, in particular, stability and robustness analysis is given in a simplified framework. The plant-model matching equation for a standard model reference controller structure is studied in a tutorial formula. Design and analysis of model reference adaptive control schemes are given for plants with relative degree 1 or larger, using a Lyapunov or gradient method based on a standard quadratic or nonquadratic cost function. For the relative degree 1 case, an  $L^{1+\alpha}$  ( $0 < \alpha < 1$ ) adaptive control design is proposed for reducing output tracking errors. An  $L^{1+\alpha}$  ( $\alpha \geq 1$ ) theory is developed for adaptive control with inherent robustness with respect to certain modeling errors. Robust adaptive control is formulated and solved in a compact framework. Assumptions on plant unmodeled dynamics are clarified, and robust adaptive laws are analyzed. Closed-loop signal boundedness and mean tracking error properties are proved. To develop adaptive control schemes without using the sign of the high frequency gain of the controlled plant, a modified controller parametrization leads to a framework of adaptive control using a Nussbaum gain for stable parameter adaptation and closed-loop stability and asymptotic output tracking.

Chapter 6 develops a model reference adaptive control theory for discrete-time linear time-invariant plants. A unique plant-model matching equation is derived, with unique controller parameters specified to ensure exact output tracking after a finite number of steps. A stable adaptive control scheme

is designed and analyzed which ensures closed-loop signal boundedness and asymptotic output tracking. It is shown that the model reference adaptive control system is robust with respect to  $L^2$  modeling errors and with modification is also robust with respect to  $L^{1+\alpha}$  ( $\alpha > 1$ ) modeling errors. Thus an  $L^{1+\alpha}$  ( $\alpha \geq 1$ ) robustness theory is developed for discrete-time adaptive control. Robust adaptive laws are derived for discrete-time adaptive control in the presence of bounded disturbances.

Chapter 7 presents two typical designs (and their analysis) of indirect adaptive control schemes: indirect model reference adaptive control and indirect adaptive pole placement control in both continuous and discrete time. Examples are used to illustrate the design procedures and analysis methods. For indirect model reference adaptive control in continuous or discrete time, a concise closed-loop error model is derived based on which the proof of signal boundedness and asymptotic output tracking is formed in a feedback and small-gain setting similar to that for the direct model reference adaptive control scheme of Chapters 5 and 6. For indirect adaptive pole placement control, a singularity problem is addressed, and closed-loop stability and output tracking are analyzed in a unified framework for both continuous and discrete time. As a comparison, a direct adaptive pole placement control scheme is presented and discussed for its potential to avoid the singularity problem.

Chapter 8 conducts a comparison study of several adaptive control schemes applied to a benchmark two-body system with joint flexibility and damping, including direct state feedback, direct output feedback, indirect output feedback, direct–indirect state feedback, and backstepping state feedback designs, with detailed design and analysis for the last two designs. With different complexity, they all ensure closed-loop signal boundedness and asymptotic output tracking. The design and analysis of the direct–indirect adaptive control scheme demonstrate some typical time-varying operations on signals in time-varying systems.

Chapter 9 first gives the design and analysis of adaptive state feedback state tracking control for multi-input systems. A multivariable state feedback adaptive control scheme is derived using LDU decomposition of a plant gain matrix. Multivariable adaptive control is applied to system identification. This chapter then develops a unified theory for robust model reference adaptive control of linear time-invariant multi-input, multi-output systems in both continuous and discrete time. Key issues such as *a priori* plant knowledge, plant and

controller parametrizations, design of adaptive laws, stability, robustness, and performance are clarified and solved. In particular, an error model for a coupled tracking error equation is derived, a robust adaptive law for unmodeled dynamics is designed, a complete stability and robustness analysis for a general multivariable case is given, and a unified multivariable adaptive control theory is established in a form applicable in both continuous and discrete time. The chapter presents some recent results in reducing *a priori* plant knowledge for multivariable model reference adaptive control using LDU parametrizations of the high frequency gain matrix of the controlled plant. Model reference adaptive control designs for multivariable systems with input or output time delays are also derived. Different adaptive control schemes, including a variable structure design, a backstepping design, and a pole placement control design for multivariable systems, are presented. Finally, robust adaptive control theory is applied to adaptive control of robot manipulator systems in the presence of parameter variations and unmodeled dynamics.

Chapter 10 presents a general adaptive inverse approach for control of plants with uncertain nonsmooth actuator nonlinearities such as dead-zone, backlash, hysteresis, and other piecewise-linear characteristics which are common in control systems and often limit system performance. An adaptive inverse is employed for cancelling the effect of an actuator nonlinearity with unknown parameters, and a linear or nonlinear feedback control law is used for controlling a linear or smooth nonlinear dynamics following the actuator nonlinearity. This chapter gives an overview of various state feedback and output feedback control designs for linear, nonlinear, single-input and single-output, and multi-input and multi-output plants as well as open problems in this area of major theoretical and practical relevance. A key problem is to develop linearly parametrized error models suitable for developing adaptive laws to update the inverse and feedback controller parameters, which is solved for various considered cases. The chapter shows that control systems with commonly used linear or nonlinear feedback controllers such as a model reference, PID, pole placement, feedback linearization, or backstepping can be combined with an adaptive inverse to handle actuator nonlinearities.

The book is focused on adaptive control of deterministic systems with uncertain parameters, dynamics and disturbances. It can also be useful for understanding the adaptive control algorithms for stochastic systems (see references for “Stochastic Systems” in Section 1.4 for such algorithms). The



material presented has been used and refined in a graduate course on adaptive control which I have taught for the past ten years at the University of Virginia to engineering, computer science, and applied mathematics students. Comments and modifications to the book can be found at

<http://www.people.virginia.edu/~gt9s/wiley-book>.

If used as a reference, this book can be followed in its chapter sequence for both continuous- and discrete-time adaptive control system design and analysis. The discrete-time contents are mainly in Sections 1.5.3 (adaptive control system examples), 2.7 and 2.8 (systems and signals), 3.6 (adaptive parameter estimation), 3.7.2 (robustness of parameter estimation), 3.8.2 (robust parameter estimation), 4.5 (state feedback adaptive control), Chapter 6 (model reference adaptive control), Sections 7.3 (indirect model reference adaptive control and adaptive pole placement control), 9.2 (multivariable model reference adaptive control), and 10.2–10.5 (adaptive actuator nonlinearity inverse control) (both in a unified continuous- and discrete-time framework). The rest of the book is for continuous-time adaptive control design and analysis.

If used as a textbook for students with knowledge of linear control systems, as a suggestion based on experience at the graduate level, the instruction may start with Sections 1.4 and 1.5 as an introduction to adaptive control (one or two lectures, 75 minutes each). Some basic knowledge of systems, signals, and stability may be taken from Sections 2.1–2.6 (system modeling, signal norms, Lyapunov stability, Gronwall-Bellman lemma, small-gain lemma, strictly positive realness and Lefschetz-Kalman-Yakubovich lemma, signal convergence lemmas including Lemmas 2.14, 2.15, and 2.16 (Barbălat lemma) for four or five lectures). Adaptive parameter estimation can be taught using Sections 3.1–3.6 in four or five lectures, including some reading assignments of robustness results from Sections 3.7 and 3.8. The design and analysis of adaptive control schemes with state feedback are presented in Sections 4.1–4.4 (three lectures), while the discrete-time results in Section 4.5 can be used as reading materials. Continuous-time model reference adaptive control in Chapter 5 can be covered in seven or eight lectures (Sections 5.1–5.5, with Section 5.6 as a reading assignment). Indirect adaptive control in Chapter 7 may need four lectures. One lecture plus reading is recommended for Chapter 8. Chapters 9 and 10 are for advanced study as either extended reading or project assignments. Further reading can be selected from the included extensive list of references on adaptive systems and control.

In this book, for a unified presentation of continuous- and discrete-time adaptive control designs in either the time or frequency domain, the notation  $y(t) = G(D)[u](t)$  (or  $y(D) = G(D)u(D)$ ) represents, as the case may be, the time-domain output at time  $t$  (or frequency-domain output) of a dynamic system characterized by a dynamic operator (or transfer function)  $G(D)$  with input  $u(\tau)$ ,  $\tau \leq t$  (or  $u(D)$ ), where the symbol  $D$  is used, in the continuous-time case, as the Laplace transform variable or the time differentiation operator  $D[x](t) = \dot{x}(t)$ ,  $t \in [0, \infty)$ , or, in the discrete-time case, as the  $z$ -transform variable or the time advance operator  $D[x](t) = x(t + 1)$ ,  $t \in \{0, 1, 2, 3, \dots\}$ , with  $x(t) \triangleq x(tT)$  for a sampling period  $T > 0$ .

Adaptive control as knowledge has no limit and as theory is rigorous. Adaptive control is a field of science. The universe is mysterious, diverse, and vigorous. The world is complicated, uncertain, and unstable. Adaptive control deals with complexity, uncertainty, and instability of dynamic systems. Taoist philosophy emphasizes simplicity, balance, and harmony of the universe. A goal of this book is to give a simplified, balanced, and harmonious presentation of the fundamentals of adaptive control theory, aimed at improving the understanding of adaptive control, which, like other control methodologies, brings more simplicity, balance, and harmony to the dynamic world.

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# Chapter 1

## Introduction

Adaptive control is based on feedback of signals in a controlled system for control adaptation to effectively handle system uncertainties. Feedback is crucial for performance guarantees in control systems, including adaptive control systems, which can be designed in either continuous or discrete time. This chapter presents some basic concepts of feedback in control systems, system modeling in continuous time and discrete time, and feedback control design and some prototypes of adaptive control systems with illustrative examples.

### 1.1 Feedback in Control Systems

A system is a set of interconnected functional components organized for certain specific tasks in a physical world. Various types of systems are all around us. A control system is a system whose behavior can be influenced by some externally acting signals. A signal which describes a system's behavior is the output of the system, while an externally acting signal is a control input to the system. There are two types of control systems: *open loop* and *closed loop* (feedback). In an open-loop control system, the input signals are prespecified, assuming an ideal situation of system operation (e.g., without any uncertainties in the system), and no system output information is used in generating the control input signal. An open-loop system is unable to adapt to system changes and is not effective for sophisticated control tasks (in control theory, an open-loop system model usually serves as a system to be controlled). A closed-loop control system utilizes its output signals for feedback to generate a control input and is much more powerful than an open-loop control system. A closed-loop control system is capable of adapting to system changes and uncertainties and achieving high performance. Almost all control systems use certain feedback and thus operate in a closed loop.